

THE PROOF OF PROPOSITION 5.3.2 IN DR. SHIRLEY'S FORMAT.

Proposition 5.3.2: For all integers $n \geq 3$, $2n+1 < 2^n$.

Proof: [By Mathematical Induction]

Let $n=3$. $2n+1 = 2 \times 3 + 1 = 7$, by substitution.
 $2^n = 2^3 = 8$, by substitution.
 $7 < 8$.

[END OF BASIS STEP]

\therefore For $n=3$, $2n+1 < 2^n$, by substitution.

Now, Suppose that k is any integer with $k \geq 3$.

Suppose that $2k+1 < 2^k$. [The Inductive Hypothesis]

[NTS: $2(k+1)+1 < 2^{(k+1)}$]

$$\begin{aligned} \therefore 2(k+1)+1 &= 2k+2+1 \\ &= (2k+1)+2 \end{aligned}$$

Since $k \geq 3$, $2 < 2^k$.

By the Inductive Hypothesis, $2k+1 < 2^k$.

$$\therefore (2k+1)+2 < 2^k + 2^k = 2 \cdot 2^k = 2^{k+1}$$

$$\therefore (2k+1)+2 < 2^{k+1}$$

$\therefore 2(k+1)+1 < 2^{k+1}$, by substitution.

\therefore By Direct Proof, For all integers k , $k \geq 3$,

If $2k+1 < 2^k$, Then $2(k+1)+1 < 2^{k+1}$.

[END OF Inductive Step].

\therefore For all integers $n \geq 3$, $2n+1 < 2^n$, by MATHEMATICAL INDUCTION.

QED

WORK SPACE:

$$\begin{aligned} 2(k+1)+1 &= 2k+2+1 \\ &= 2k+3 \\ &= (2k+1)+2 \\ &< 2^k + 2 \\ &< 2^k + 2^k \\ &= 2 \cdot 2^k \\ &= 2^{k+1} \end{aligned}$$